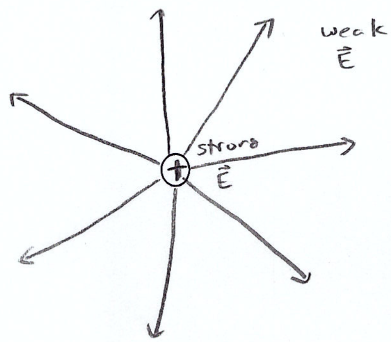
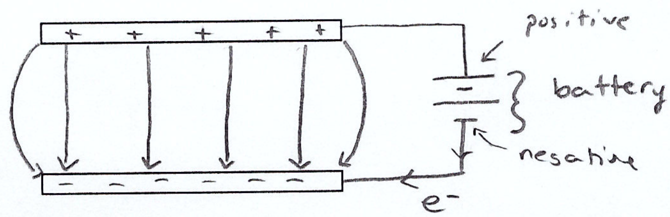
Electric Potential in a Uniform Field: Parallel Plates

- Compare the electric field around a point charge and the electric field between two parallel plates





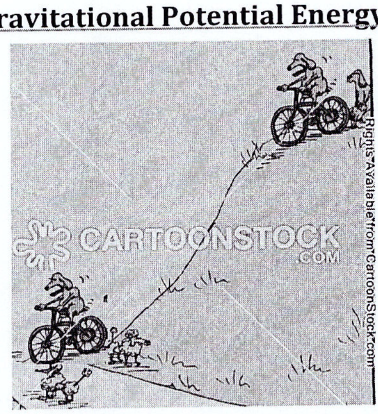
use $\vec{E} = \frac{kq}{r^2}$
b/c electric field strength depends on distance

uniform electric field b/w parallel plates
∴ NEVER use $\vec{E} = \frac{kq}{r^2}$!

  - * Parallel plates are an example where the electric field strength is uniform
  - We need a different equation to deal with this uniform electric field strength because so far our equations deal with a changing electric field strength
- To help us understand the equations used for a uniform electric field, you first need to understand the electric potential energy that occurs within the parallel plates
- Electric potential energy is very similar to gravitational potential energy
  - A charged object inside a uniform field can have high electric potential energy and low electric potential energy
  - The law of conservation of energy applies and electrical potential energy can be converted into kinetic energy and vice versa

Gravitational Potential Energy (E_p)

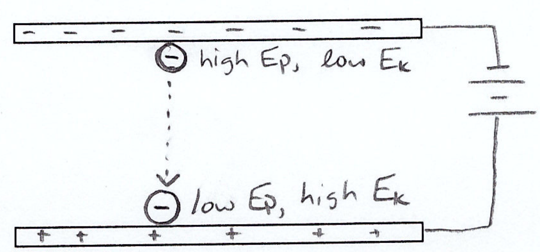
high E_p, low E_k.



low E_p, high E_k

$\Delta E_p \xrightarrow{\text{converted to}} \Delta E_k$

Electric Potential Energy (E_p)



$\Delta E_p \xrightarrow{\text{converted to}} \Delta E_k$

* repulsion = high E_p
attraction = low E_p

  - Therefore uniform electric fields/parallel plates are a method to <u>*accelerated/decelerate*</u> charged particles.

- The change in the electric potential energy per unit charge is defined as the **electric potential difference** (or **electric potential** or **voltage**) and can described by the following equation:

$$\Delta V = \frac{\Delta E}{q}$$

where ΔV is potential difference in units of volts (V)
ΔE is the change in energy in joules (J) * not electric field ($\vec{E}$)
q is charge in coulombs (C)

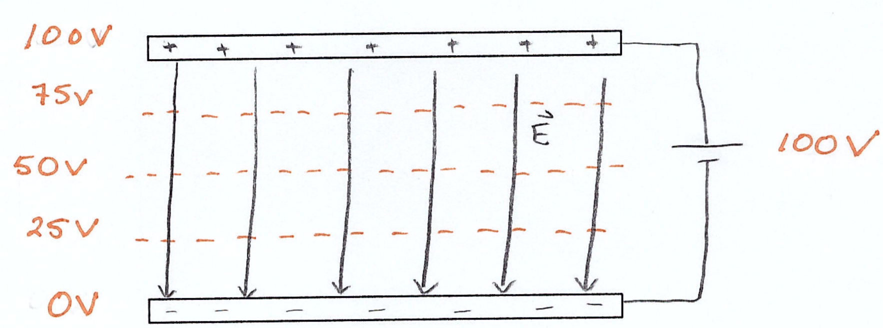
  - Potential difference is a scalar quantity
  - Small amounts of energy can also be measured in units of **electron volts (eV).** However, joules are the SI units for energy, so all calculations involving energy in formulas need to be done in joules!

$$1 \text{ eV} = 1.6 \text{x} 10^{-19} \text{ J}$$ on data sheet!

EXAMPLE: Convert 2.10eV into joules.

$$2.10 \text{ eV} \times \left(\frac{1.6\times10^{-19}\text{ J}}{1\text{eV}}\right) = \boxed{3.36\times10^{-19}\text{ J}}$$

* - Since the electric potential energy is changing between a set of parallel plates based on electrostatic repulsion/attraction, the *electric potential* is also *changing*, even though the *electric field remains uniform*



- Even though the electric field between a set of parallel plates is uniform, the electric field strength can be altered by either changing the separation distance of the plates or the electric potential between the plates as shown by the following equation:

$$\vec{E} = \frac{\Delta V}{d}$$

where $\vec{E}$ is electric field strength (N/C or V/m)
ΔV is potential difference between plates (V)
d is distance between plates (m)

EXAMPLES:

1. An electric field of 3000V/cm may cause the air to ionize and allow electrons to jump from one surface to another causing a spark. An electrostatic charge is produced on a student walking across a rugged floor. He reaches for a doorknob. Assuming an electric field of 3000V/cm, determine the electric potential difference required between the finger and doorknob to produce a spark when they are separated by 1.2cm.

$\vec{E} = 3000\,V/cm$

$d = 1.2\,cm$

$\Delta V = ?$

$$\vec{E} = \frac{\Delta V}{d} \Rightarrow \vec{E}d = \Delta V$$

$$\Delta V = (3000\,V/cm)(1.2\,cm) = 3600\,V$$

$$\boxed{\Delta V = 3.6 \times 10^{3}\,V}$$

$q = 1.6\times10^{-19}\,C$

$m = 9.11\times10^{-31}\,kg$ from data sheet!

2. An electron has a speed of 5.80×10^{5}m/s when it enters a set of parallel plates. If the electric potential difference between the plates is 140V, what is the exit speed of the electron?

$V_i = 5.80\times10^5\,m/s$

$\Delta V = 140V$

$V_f = ?$

(2) $E_k = \frac{1}{2}mv^2$

$\Delta E_k = E_{kf} - E_{ki}$

(1) $\Delta V = \Delta E / q$

(1) $\Delta E = \Delta V q = (140V)(1.6\times10^{-19}C) = 2.24\times10^{-17}\,J$

(2) $\Delta E = E_{kf} - E_{ki} \Rightarrow E_{kf} = \Delta E + E_{ki} = \Delta E + \frac{1}{2}m(V_i)^2$

$E_{kf} = 2.24\times10^{-17}\,J + \frac{1}{2}(9.11\times10^{-31}\,kg)(5.80\times10^5\,m/s)^2$

$E_{kf} = 2.24\times10^{-17}\,J + 1.532302\times10^{-19}\,J$

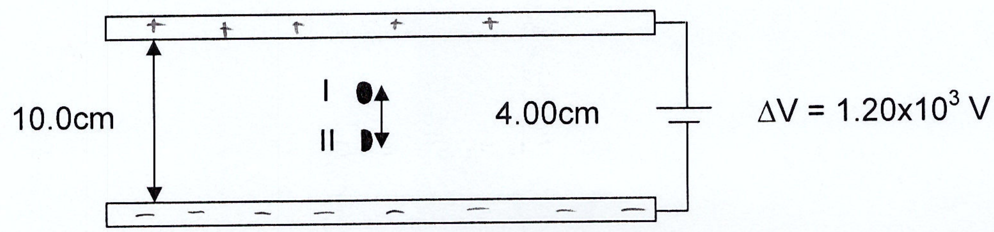
$E_{kf} = 2.255...\times10^{-17}\,J$

$E_{kf} = \frac{1}{2}m(v_f)^2 \Rightarrow V_f = \sqrt{\frac{2E_k}{m}}$

$V_f = \sqrt{\frac{2(2.255...\times10^{-17}J)}{9.11\times10^{-31}kg}} = 7.0365...\times10^6\,m/s$

$\boxed{V_f = 7.04\times10^6\,m/s}$

* 3. The diagram below shows two parallel plates separated by 10.0cm. Calculate the change in voltage between points I and II when they are 4.00cm apart.



unifor $\vec{E}$, but changing ΔV!

B/w Plates

$\Delta V_1 = 1.20 \times 10^3$ V

$d_1 = 0.100$ m

Within Plates

$\Delta V_2 = ?$

$d_2 = 0.0400$ m

1

$\vec{E} = \frac{\Delta V_1}{d_1}$

B/w plates

2

$\vec{E} = \frac{\Delta V_2}{d_2}$ within plates

1 $\vec{E} = \frac{\Delta V_1}{d_1} = \frac{1.20 \times 10^3 V}{0.100 m} = 12000 N/C$

2 $\vec{E} = \frac{\Delta V_2}{d_2} \Rightarrow \Delta V_2 = \vec{E} d_2$

$\Delta V_2 = (12000 N/C)(0.0400 m) = 480$ V

$\boxed{\Delta V_2 = 480 V}$

Now try pg. 108 #1, 5, 8(a,c), 9, 11, 18, 19, 25, 28, 34-37 & Practice Problems #1-5

Practice Problems

1. A cell membrane is 1.0×10^{-7}m thick and has an electric potential difference between its surfaces of 0.070V. What is the electric field within the membrane? **[7.0×10^{5} V/m]**

2. How much electrical potential energy (in eV) does an alpha particle gain when it moves between two oppositely charged parallel plates with a voltage of 20000V? **[4.00×10^{4} eV]**

3. An electron starts from rest and accelerates across two parallel plates to obtain a kinetic energy of 9.6×10^{-17} J.
   a. Determine the potential difference between the plates. **[600V]**
   b. Determine the electric potential energy the electron initially possessed. **[9.6×10^{-17} J]**
   c. Determine the maximum speed of the electron. **[1.45×10^{7} m/s]**

4. What potential difference is needed to decelerate an alpha particle from 1.40×10^{6} m/s to 6.80×10^{5} m/s. **[1.56×10^{4} V]**

5. Two parallel plates are separated by a distance of 3.75cm. Two points, A and B, lie along a perpendicular line between the parallel plates and are 1.10cm apart. Between the two points, there is a difference in electric potential of 6.00V. Calculate the electric potential between the parallel plates. **[20.5 V]**

6. The cell membrane may be thought of as a set of charged parallel plates. The electric potential difference between the outside and the inside of the membrane is about 0.70V and the thickness of the membrane is 5.0×10^{-9}m. If 0.18eV is used to move a Na^{+} ion (has +1 elementary charge), how far does the Na^{+} move across the membrane? **[1.3×10^{-9} m]**

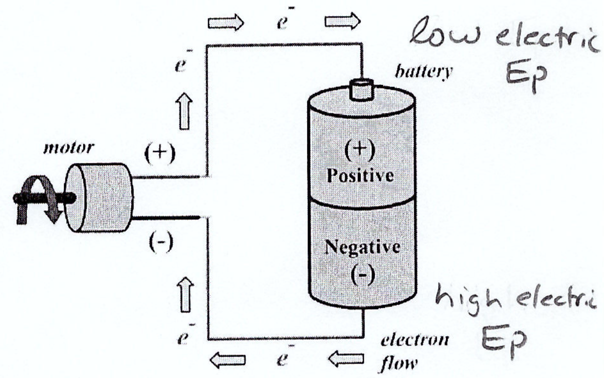
Show that a $N/C = V/m$

$$\frac{\cancel{V}}{m} \times \left(\frac{J/C}{\cancel{V}}\right) = \frac{\cancel{J}}{m \cdot C} \times \left(\frac{kg \cdot m^2/s^2}{\cancel{J}}\right) = \frac{kg \cdot m^{\cancel{2}}/s^2}{\cancel{m} \cdot C}$$

$$= \frac{kg \cdot m/s^2}{C}$$ but $F = ma$ $\therefore$ $$\frac{\cancel{kg \cdot m/s^2}}{C} \times \left(\frac{N}{\cancel{kg \cdot m/s^2}}\right)$$

$$= N/C$$

- A battery is a source of potential difference
  - Just as a water pump can increase the gravitational potential energy of water, a battery will increase the electrical potential energy of a charge
  - Electric potential is the "pushing power" behind charges; pushing charges to a higher electric potential energy



- When a potential difference is applied to a circuit, a *current* is created. **Current (*I*)** is the flow rate of electric charge
  - EXAMPLE: A battery is a voltage source that causes a current to flow through a mp3 player and produces the music you hear
  - The definition of current is also expressed as an equation

$$I = q/t$$

where I is current (C/s or A)
q is charge (C) total charge!
t is time (s)

  - **Ampere (amps or A):** fundamental unit of current

if 5000 electrons passed through a point in 8.2 ms, what is the current?

$$I = q/t = \frac{5000(1.6\times10^{-19}\,C)}{8.2\times10^{-3}\,s} = 9.756...\times10^{-14}$$

$$\boxed{I = 9.8\times10^{-14}\,A}$$

$q = 1.6\times10^{-19}\,C$

$m = 1.67\times10^{-27}\,kg$

4. A proton starts from rest and accelerates across a potential difference of 200V between two charged plates that are 20 cm apart.
   a) Determine the maximum speed of the proton.
   b) Determine the current.

a.) $\Delta V = 200\,V$

$V_i = 0.0\,m/s$

$d = 20cm \times \left(\frac{10^{-2}m}{1cm}\right)$

$d = 0.20\,m$

$V_f = ?$

$E_k = \frac{1}{2}mv^2$

$E_{k_f} - E_{k_i} = \Delta E_k$ ②

$\Delta V = \Delta E / q$ ①

① $\Delta E = \Delta V q = (200V)(1.6\times10^{-19}C)$

$\Delta E = 3.2\times10^{-17}\,J$

② $\Delta E_k = E_{k_f} - \cancel{E_{k_i}}^{\,0} = \frac{1}{2}m(v_f)^2$

$\therefore\ V_f = \sqrt{\frac{2\Delta E_k}{m}} = \sqrt{\frac{(3.2\times10^{-17}J)2}{1.67\times10^{-27}kg}}$

$V_f = 1.9576...\times10^{5}\,m/s$

$\boxed{V_f = 2.0\times10^{5}\,m/s}$

b.) $I = q/t$ ②

$d = \left(\frac{V_f + V_i}{2}\right)t$ ①

① $d = \left(\frac{V_f + \cancel{V_i}^{\,0}}{2}\right)t \Rightarrow t = \frac{2d}{V_f} = \frac{2(0.20m)}{1.9576...\times10^{5}m/s}$

$t = 2.0432...\times10^{-6}\,s$

② $I = q/t = \frac{(1)(1.6\times10^{-19}c)}{2.0432...\times10^{-6}s} = 7.8305...\times10^{-14}\,A$

$\boxed{I = 7.8\times10^{-14}\,A}$

$q = 1.6\times10^{-19}\,C$

$m = 1.67\times10^{-27}\,kg$

uniform field

5. A proton is placed in an electric field between two parallel plates. If the plates are 6.0 cm apart and have a potential difference between them of 7.5x10[1] V, how much work is done against the electric field when the proton is moved 3.0 cm parallel to the electric field?

work is being done!
b/c Fe is causing the $\vec{d}$!

$\Rightarrow W = Fd\cos\theta$ but $\cos 0° = 1$
$\therefore W = Fd$

$W = \Delta E = ?$

$\Delta V = 7.5\times10^{1}\,V$

$d_{plates} = 6\,cm = 0.060\,m$

$d_{particle} = 3.0\,cm = 0.030\,m$

$W = F d_{particle}$ (3)

$\vec{E} = \frac{\vec{F_e}}{q}$ (2)

$\vec{E} = \frac{\Delta V}{d_{plates}}$ (1)

(1) $\vec{E} = \frac{\Delta V}{d_{plates}} = \frac{7.5\times10^{1}\,V}{0.060\,m} = 1250\,V/m$

(2) $\vec{E} = \frac{\vec{F_e}}{q} \Rightarrow \vec{F_e} = \vec{E}q = (1250\,V/m)(1.6\times10^{-19}\,C) = 2.0\times10^{-16}\,N$

(3) $W = Fd_{particle} = (2.0\times10^{-16}\,N)(0.030\,m) = 6.0\times10^{-18}\,J$

$\boxed{W = 6.0\times10^{-18}\,J}$

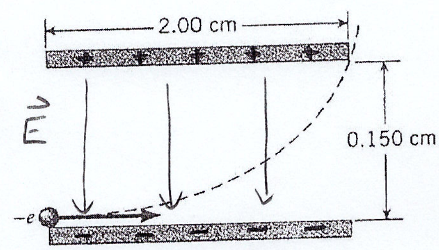
OR $6.0\times10^{-18}\,J \times \left(\frac{1\,eV}{1.6\times10^{-19}\,J}\right) = \boxed{38\,eV}$

* explain how students could use $\Delta V = \frac{\Delta E}{q}$ correctly for this problem!

*****Now try pg. 108 # 6, 7 & pg. 115 #30, 33 & Practice Problem #6*****

- In the previous examples, we only considered problems where the charged particles are moving *parallel* to the field. Therefore, if force and distance are parallel to each other, work is being done and the particles accelerate.
  - Recall though that if a particle is moving *perpendicular* to the force, no work is being done and therefore the particles maintain uniform motion

EXAMPLE: The drawing shows an electron entering the lower left side of two parallel plates and exiting at the upper right side. The initial speed of the electron is 7.00×10^6 m/s. The parallel plates are 2.00cm long and separated by 0.150cm. Calculate the electric field strength inside the parallel plates.



* similar to projectile motion *

Horizontal / Perpendicular
(uniform motion)

$v = \frac{d}{t}$

$v = 7.00\times10^6$ m/s
$d = 0.02$ m
$t =$

equal

Vertical / Parallel
(accelerated)

$d = 0.0015$ m
$v_i = 0.0$ m/s
$t =$
$a = ?$

① $v = \frac{d}{t} \Rightarrow t = \frac{d}{v}$

$t = \frac{0.02\text{ m}}{7.00\times10^6\text{ m/s}}$

$t = 2.857\ldots\times10^{-9}$ s

② $d = \cancel{v_i t} + \frac{1}{2}at^2$
0

$\frac{2d}{t^2} = a = \frac{(2)(0.0015\text{ m})}{(2.857\ldots\times10^{-9}\text{ s})^2} = 3.675\times10^{14}$ m/s²

③ $F_{net} = ma = F_e + (-F_g)$
0 * negible for atomic particles

$\therefore F_e = ma = (9.11\times10^{-31}\text{ kg})(3.675\times10^{14}\text{ m/s}^2)$
$F_e = 3.3479\ldots\times10^{-16}$ N

④ $\vec{E} = \frac{F_e}{q} = \frac{(3.3479\ldots\times10^{-16}\text{ N})}{1.6\times10^{-19}\text{ C}} = 2.092\ldots\times10^3$ N/C

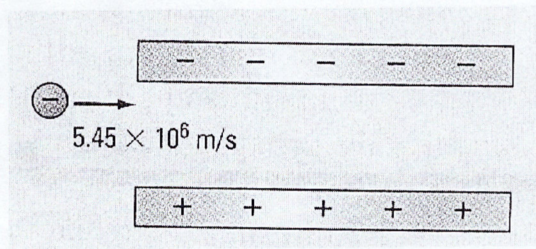
$\therefore \boxed{\vec{E} = 2.09\times10^3 \text{ N/C, down}}$

*****Now try pg. 115 # 31-32 and Practice Problems*****

Practice Problems

1. An electron, travelling at 2.3×10^3 m/s, enters perpendicular to the electric field between two horizontal charged parallel plates. If the electric field strength is 1.5×10^2 V/m, calculate the time taken for the electron to deflect a distance of 1.0×10^{-2} m toward the positive plate. **[2.8×10^{-8} s]**

2. An electron, travelling horizontally at a speed of 5.45×10^6 m/s, enters a parallel plate capacitor with an electric field of 125N/C between the plates, as shown in the figure.



   a. Sketch the electric field lines between the plates and the motion of the electron through the capacitor.
   b. Calculate the acceleration of the electron. **[2.20×10^{13} m/s^2, down]**
   c. If the electron falls a vertical distance of 6.20×10^{-3} m toward the positive plate, how far will the electron travel horizontally between the plates? **[0.130m]**

3. A proton is fired parallel in between a set of parallel plates at a height that is exactly midway between the set of plates with an initial speed of 2.50×10^6 m/s, as shown in the diagram below. The electric potential difference between the plates is 175 V. If the proton travelled a total distance of 15.0cm parallel to the plates, what is the separation distance between the plates? **[0.777 cm]**