Rearrange/Solving Equations

When we are given an equation or formula, written in mathematical symbols (algebra), we are normally asked to find one value/symbol. We isolate the variable by doing mathematical functions to both sides of the equation (+, -, x, ÷).

Steps:

1. Identify the unknown you are trying to isolate.
   Ex. $3x^2 + 9 = 12$

2. Isolate the "chunk" with the unknown doing the opposite math function on both sides.
   Ex. $3x^2 + 9 = 12 - 9$
   -9

   $3x^2 = 3$

3. Multiply, divide, square root, square, etc. to do the opposite math function on both sides to isolate the unknown completely.
   Ex.
   $\frac{3x^2}{3} = \frac{3}{3} \Rightarrow \sqrt{x^2} = \sqrt{1} \Rightarrow \boxed{x = 1}$

Note: If the unknown is on the bottom, multiply by the unknown on both sides first to move it to the top.

<u>Examples:</u> Solve for the unknown.

1. $a_c = \frac{4\pi^2 r}{T^2}$ for T

$a_c(T^2) = 4\pi^2 r$

$\frac{a_c(T^2)}{a_c} = \frac{4\pi^2 r}{a_c}$

$\sqrt{T^2} = \sqrt{\frac{4\pi^2 r}{a_c}}$

$\boxed{T = \sqrt{\frac{4\pi^2 r}{a_c}}}$

2. $d = v_i t + \frac{1}{2} a t^2$ for v_i

$d - \frac{1}{2}at^2 = v_i t + \frac{1}{2}at^2 - \frac{1}{2}at^2$

$\frac{d - \frac{1}{2}at^2}{t} = \frac{v_i t}{t}$

$\boxed{\frac{d - \frac{1}{2}at^2}{t} = v_i}$

*** Now try pg. 9 #1 & Practice Problems ***

Practice Problems

1. $v = \frac{d}{t}$ for t
2. $a = \frac{(v_f - v_i)}{t}$ for v_f
3. $d = v_i t + \frac{1}{2}at^2$ for a
4. $T = 2\pi\sqrt{(l/g)}$ for l
5. $E_k = \frac{1}{2}mv^2$ for v
6. $F_c = \frac{mv^2}{r}$ for v
7. $v_f^2 = v_i^2 + 2ad$ for d
8. $F_c = \frac{4\pi^2 mr}{T^2}$ for T
9. $v_f^2 = v_i^2 + 2ad$ for v_i
10. $d = \frac{1}{2}(v_i + v_f)t$ for v_f

1.) $t = \frac{d}{v}$

2.) $v_f = at + v_i$

3.) $\frac{2(d - v_i t)}{t^2} = a$

4.) $\left(\frac{T}{2\pi}\right)^2 g = l$

5.) $\sqrt{\frac{2E_k}{m}} = v$

6.) $v = \sqrt{\frac{F_c r}{m}}$

7.) $\frac{(v_f^2 - v_i^2)}{2a} = d$

8.) $T = \sqrt{\frac{4\pi^2 mr}{F_c}}$

9.) $\sqrt{v_f^2 - 2ad} = v_i$

10.) $v_f = \frac{2d}{t} - v_i$

Significant Digits and Scientific Notation

- **Significant Digits**: the digits we know for certain or precisely **plus** an estimated digit which accounts for errors

General Rules and Dealing with Zeros

- All digits 1-9 count as a significant digit
- Leading zeros are **NOT** significant digits

0.00274   3 sig. figs.

- Trailing zeros (to the right of the decimal) **DO** count as significant digits

0.72600   5 sig. figs.

- **ONLY** round to the answer to correct significant digits when the entire problem is solved
- Significant digits is your **very last** step in completing all calculations

Addition and Subtraction Rules

- Out of all the numbers being added/subtracted, the number that has the *least amount of decimal places* (ie. least precise) determines the number of decimal places in your answer
- When adding/subtracting, the significant digits are determined only by the *number of decimal places*

$$\begin{array}{r} 274.0271 \\ 10.20 \\ \hline 284.2271 \end{array}$$

← least decimal places

∴ answer has 2 decimal places

$\boxed{284.23}$ recorded answer

Multiplication and Division Rules

- Count the number of significant digits in all the numbers that are being multiplied/divided
- The number with the *least amount of significant digits* (ie. least precise value) determines the number of significant digits your answer needs to have

$(22.50) \times (0.0970) = 2.1825$

4 sig. figs

3 sig. figs
↓
least # of sig. figs ∴ answer to 3 sig. figs

$\boxed{2.18}$

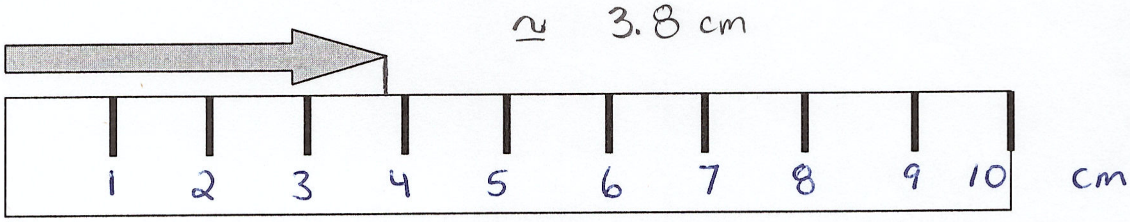
Exact Number

- Exact numbers (ie. 800 people) are counted and defined numbers, so they are not considered when determining the significant digits of the answer, even if the number was used in the calculations

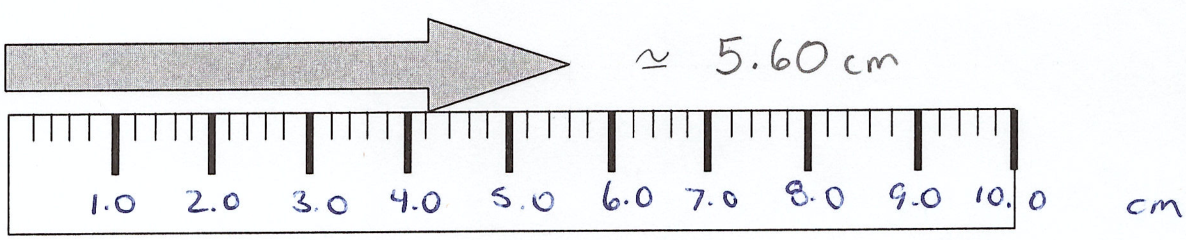
Significant Digits on Rulers

- Example: Record the length to the correct significant digits

≃ 3.8 cm



≃ 5.60 cm



* for sig. figs, the # we know for sure + a guess to the next place value

Scientific Notation

- For bigger and smaller numbers, it is more convenient to record the answer in *scientific notion*
- Also helps to maintain the correct significant digits

- Example: 5236978000 km needs to be recorded to three significant digits. Record this distance in scientific notation.

$$5.24 \times 10^{9} \text{ km}$$

3 sig. figs.

*****Now try pg. 3 #1-4*****

Unit Analysis and Conversions

- It is quite often you will be required to convert one unit into another
- Unit analysis is a mathematical technique used to convert units
  - Multiply/divide to cancel out the unwanted units and convert into the wanted units
- Notice the information on the data sheet

small	nano	n	10^{-9}	
	micro	μ	10^{-6}	
	milli	1 m =	10^{-3}	g
	centi	c	10^{-2}	
	deci	d	10^{-1}	
	deka	da	10^{1}	
	hecto	h	10^{2}	
	kilo	1 k =	10^{3}	m
	mega	M	10^{6}	
big	giga	G	10^{9}	

referring to any standard unit (ie. L, g, s, m)

Examples

1. Convert 5500 mm into km.

$$5500 \cancel{\text{mm}} \times \left(\frac{10^{-3} \cancel{\text{m}}}{1 \cancel{\text{mm}}}\right) \times \left(\frac{1 \text{ km}}{10^{3} \cancel{\text{m}}}\right) = 0.005500 \text{ km}$$

or

$$5.500 \times 10^{-3} \text{ km}$$

2. Convert 20 km/h to m/s.

$$\frac{20\ \cancel{km}}{1\ \cancel{h}} \times \left(\frac{10^3\ m}{1\ \cancel{km}}\right) \times \left(\frac{1\ \cancel{h}}{3600\ s}\right) = 5.\overline{5}\ m/s$$

$$\boxed{5.6\ m/s}$$

3. Convert 720000 s into years.

$$720000\ \cancel{s} \times \left(\frac{1\ \cancel{hr}}{3600\ \cancel{s}}\right) \times \left(\frac{1\ \cancel{day}}{24\ \cancel{hr}}\right) \times \left(\frac{1\ year}{365\ \cancel{days}}\right)$$

$$= \boxed{0.022831\ years}$$

*****Now try pg. 6 #2b, c, e, f, h, j, 4a, c, d*****

*** Review Assignment ***